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Transport properties and localized edge modes arising at imperfect kagome lattices

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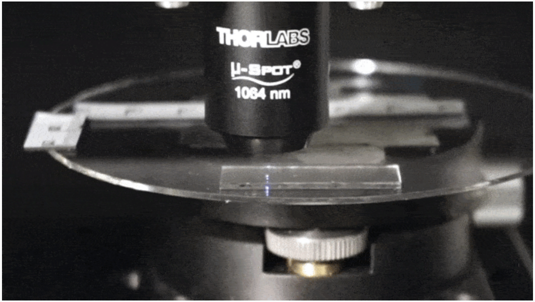
Photonic lattice model and femtosecond laser writing technique

From Maxwell's equations, the following discrete Schrödinger-like equations are derived [1]:

$$-i\frac{\partial u_{\vec{n}}}{\partial z} = \sum_{\vec{m}\neq\vec{n}} V_{\vec{n},\vec{m}} u_{\vec{m}} \tag{1}$$

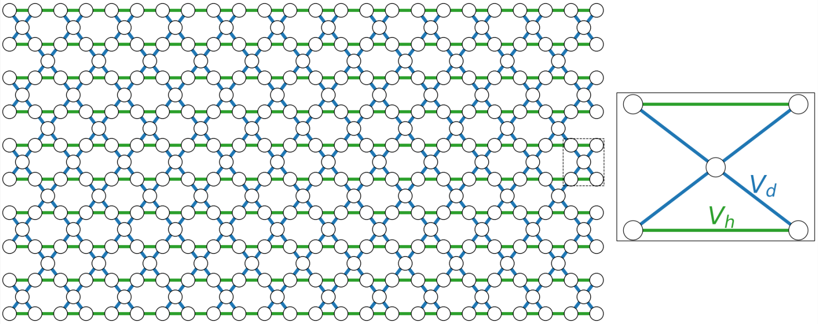
Here, $u_{\vec{n}}$ is the field amplitude in the waveguide at $\vec{n}$ position, z is the propagation distance (depth of the waveguide) and $V_{\vec{n},\vec{m}}$ is the coupling matrix element.



By focusing a femtosecond laser beam on silica glass we slightly modify the local refraction index, inducing waveguiding [2].

Perfect photonic kagome lattice

Building upon this lattice we study several variations (imperfections) by removing certain waveguides.



We consider nearest neighbor coupling (V_d and V_h) only.

Stationary solutions

- First, we use an ansatz $\vec{u}(z) = \vec{u}_\lambda e^{i\lambda z}$ in the equivalent differential vector equation $-i\frac{\partial \vec{u}}{\partial z} = \hat{V}\vec{u}$.
- Then, we solve the eigenvalue problem obtaining the system eigenmodes $\vec{u}_\lambda$.
- Finally, we calculate the participation ratio $R(\lambda) = \frac{(\sum_{\vec{n}} |u_{\lambda,\vec{n}}|^2)^2}{\sum_{\vec{n}} |u_{\lambda,\vec{n}}|^4}$, where $u_{\lambda,\vec{n}}$ is the eigenvector $\vec{u}_\lambda$ component with eigenfrequency λ that is in the $\vec{n}$ lattice position.
- We compare two different kagome lattice variations in the isotropic regime ($V_d = V_h$).

Corner and lateral defects

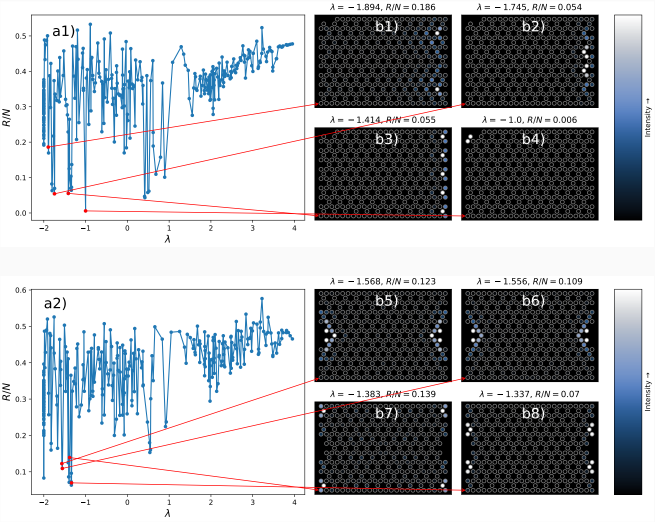


Figure 1: a1)-a2) Normalized participation ratio v/s eigenfrequency λ for photonic kagome lattices with corner and lateral defects eigenmodes. b1)-b8) Selected localized eigenmodes. The brighter (darker) the waveguide color, the higher (lower) intensity $|u_{\vec{n}}|^2$ the light has.

Eigenmode dynamics

We analyze the transverse dynamic of light (by solving Eq.1 numerically) based on the superposition of two localized eigenmodes that arise in kagome lattices with lateral defects, in particular, the ones seen in Fig. 1b6) and Fig. 1b7).

The average horizontal lattice position of light can be defined as $\langle x \rangle = \sum_{\vec{n}} |u_{\vec{n}}|^2 x_{\vec{n}} / \sum_{\vec{n}} |u_{\vec{n}}|^2$. Where $u_{\vec{n}}$ describes the amplitude of the electric field located at lattice site $\vec{n}$ and $x_{\vec{n}}$ represents the horizontal position of the lattice site.

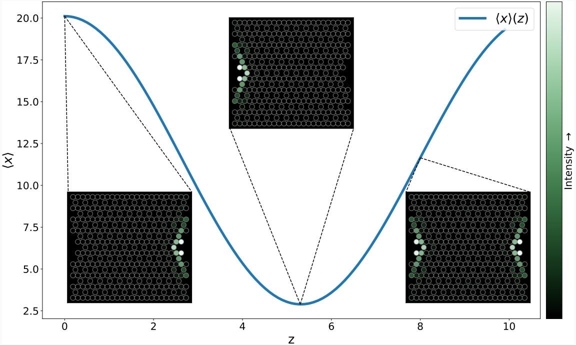


Figure 2: Evolution of $\langle x \rangle$. Inset figures show the position of light in the lattice at different values of z. To construct this figure, a total propagation distance of $z_{max} = 10$ and coupling values of $V_d = V_h = 50$ were used.

Two and three guide excitation

To study this phenomena with a simpler initial condition, we analyze the dynamic by exciting two and three sites, while defining the same propagation distance and couplings as before.

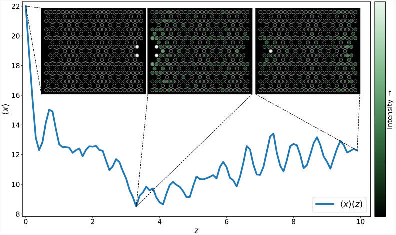


Figure 3: Light dynamics exciting two guides.

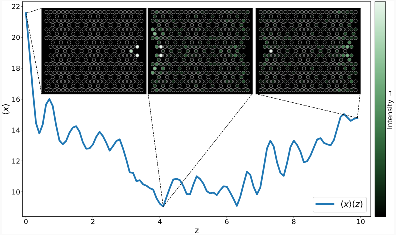


Figure 4: Light dynamics exciting three guides.

A simple defect in the corner of a kagome lattice has strong effects on transport properties. For that imperfection we observe localization-delocalization transitions as a function of phase difference between the initial excited sites [3]: Considering an initial condition as seen in Fig. 5a, we solve the dynamics of the system for couplings $V_d/V_h \in [0, 2]$ and for phase differences $\phi \in [0, 2\pi]$.

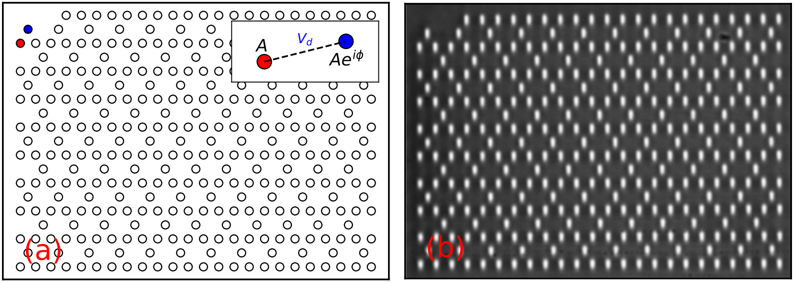


Figure 5: (a) Kagome lattice simulation with a corner defect. The two colored sites in the corner denotes the initial condition: Red site with amplitude A and blue site with amplitude $Ae^{i\phi}$, where ϕ is the phase difference between site amplitudes. (b) Experimentally fabricated kagome lattice with the same corner defect.

Localization-delocalization transitions

We compute the normalized output participation ratio R/N as function of ϕ and V_d/V_h. As seen in Fig. 6a, we observe a geometric zone (rotated ellipse-like) of protected information: inside we have high localized propagation ($R/N \approx 0.02$), and outside we have dispersion. An interesting case is when $V_d = V_h$ (see Fig.6b), as this transition is strong and highly symmetric: When $\phi = \{0, 2\pi\}$ we have great dispersion ($R/N \approx 0.45$, see Fig.6b2), and when $\phi = \pi$, light stays in the first excited sites ($R/N = 0.02$, see Fig.6b1).

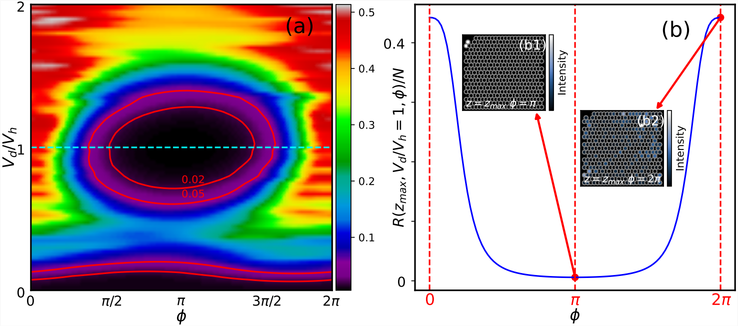


Figure 6: (a) Output participation ratio R/N as function of ϕ and V_d/V_h. Red lines denote zones where $R/N = 0.02$ and = 0.05. Dashed line denotes the isotropic case. (b) Output participation ratio when $V_d = V_h$, (b1) shows output of the system when $\phi = \pi$, and (b2) when $\phi = 2\pi$.

Imperfections can give rise to new transport properties in photonic kagome lattices. In particular, localized eigenmodes that appear thanks to the removal of certain waveguides can be combined in order to generate side-to-side light oscillations or can be directly studied with the aim of observing localization-delocalization transitions.

Acknowledgments and references

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